Inducing risk preferences in economics experiments

Ian M. Dobbs and Anthony D. Miller*

Newcastle University Business School, Armstrong Building, Victoria Road, Newcastle upon Tyne NE1 7RU, UK

*Corresponding Author. E-mail: a.d.miller@ncl.ac.uk

A common procedure in experiments is to use binary lotteries to induce in all subjects pre-specified risk preferences. The validity of this procedure has been established only for a subject performing a single task, yet the procedure is normally applied in multi-task settings. This article formally analyses the multi-task case and establishes necessary and sufficient conditions relating to experimental design. New guidance is provided for the design of experiments involving interdependent tasks.

1. Introduction

The Risk-Preference Inducing Procedure (*RPIP*) uses binary lotteries to induce risk preferences in experimental work. Its validity was established by Berg *et al* (1986) for a subject performing a single task.[1] In practice, subjects are usually required to perform a series of tasks. The present article addresses the validity of *RPIP* in this setting. It shows that the assumptions underlying the special case in Berg *et al* (1986) are no longer sufficient for the validity of *RPIP*, and must be supplemented by an additional restriction relating to independence of stages. Many experiments will not satisfy this condition. We also show that a simple modification of the *RPIP* procedure suffices to restore its validity in such cases.

2. The Analytic Framework, *RPIP* and Bundling

In order to introduce the notation, it is useful to review the single-task problem, as analysed in Berg *et al* (1986).

Single-Task single-subject experiments with RPIP

The subject chooses an action a from a set of actions, $a \in A$. Suppose the subject then receives a random payment x from a finite set of rewards, $x \in X$, with conditional probability density $g(x \mid a)$. Assuming the subject's preferences can be represented by a von Neumann-Morgenstern (*VNM*) utility function $U(x)$, the decision problem is:

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[1] Berg *et al* (1986) suggest that *RPIP* '…can be utilized in any experimental setting', but this has not to date been formally demonstrated. The present article shows that the claim needs to be qualified.

$$\max_{a \in A} \int_{x \in X} g(x \mid a) U(x) dx \tag{1}$$

X and $g(x \mid a)$ will normally be specified in the experimental design. But since $U(x)$ is not known by the experimenter, the predicted behaviour of the subject, his chosen a, is indeterminate. If instead, *RPIP* is applied, then the subject is rewarded in an 'experimental currency', exchanged later for probability points in the play of a subsequent two-prize lottery. Denote the random reward of 'experimental currency' by $q \in Q$ with conditional density $f(q \mid a)$, where Q is a closed bounded interval of the set of real numbers. Choose for the binary lottery two money prizes, $(\bar{x}, \underline{x})$, with $\bar{x} > \underline{x}$ so that $U(\bar{x}) > U(\underline{x})$, and a function $P(\bar{x} \mid q) = 1 - P(\underline{x} \mid q) = G(q)$ for exchanging q into probability points for the binary lottery, then the subject's decision problem is now

$$\max_{a \in A} \int_{q \in Q} f(q \mid a) \{ P(\bar{x} \mid q) U(\bar{x}) + (1 - P(\bar{x} \mid q)) U(\underline{x}) \} dq \tag{2}$$

$$= \max_{a \in A} \int_{q \in Q} f(q \mid a) \{ G(q) U(\bar{x}) + (1 - G(q)) U(\underline{x}) \} dq \tag{3}$$

$$= \max_{a \in A} \left(\int_{q \in Q} f(q \mid a) G(q) [U(\bar{x}) - U(\underline{x})] dq + \int_{q \in Q} f(q \mid a) U(\underline{x}) dq \right) \tag{4}$$

$$= \max_{a \in A} \left([U(\bar{x}) - U(\underline{x})] \int_{q \in Q} f(q \mid a) G(q) dq + U(\underline{x}) \int_{q \in Q} f(q \mid a) dq \right) \tag{5}$$

Since $\int_{q \in Q} f(q \mid a) dq = 1$, Equation 5 reduces to

$$\max_{a \in A} \left(U(\underline{x}) + [U(\bar{x}) - U(\underline{x})] \int_{q \in Q} f(q \mid a) G(q) dq \right) \tag{6}$$

Since $U(\bar{x}) > U(\underline{x})$ in (6), the optimal solution, a^* can thus be written as

$$a^* = \arg \max_{a \in A} \left(\int_{q \in Q} f(q \mid a) G(q) dq \right) \qquad (7)$$

That is, the subject behaves as if maximising the expected value of $G(q)$, a function chosen by the experimenter. By choosing a convex, linear or concave increasing function $G(q)$, respectively, risk-seeking, risk-neutral or risk-averse preferences are induced. This is the basic single-task result obtained in Berg *et al* [1986].

Sequential multi-task single-subject experiments

However, the experimental norm involves a multi-task environment, and this has not received a formal treatment to date. Consider then a subject performing a sequence of tasks where the choice at the ith stage is denoted $a_i \in A_i$, $(i = 1,...,n)$. In order to control risk preferences at each stage, each decision is given a separate reward of experimental currency $q_i \in Q_i$, $(i = 1,...,n)$. All of these features are assumed given in any experimental design that incorporates use of *RPIP*. Let $\mathbf{a} = (a_1,...,a_n)$, $A = A_1 \otimes ... \otimes A_n$ and $\mathbf{q} = (q_1,...,q_n)$, with conditional density function

$$f(\mathbf{q} \mid \mathbf{a}) = f_1(q_1 \mid a_1) \times f_2(q_2 \mid a_2, q_1, a_1) \times ... \times f_n(q_n \mid a_n, q_{n-1}, a_{n-1},..., q_1, a_1) \quad (8)$$

Equation 8 allows for interdependence between stages, whilst recognising the sequential nature of stages. Thus the density function for q_1 may *only* depend on a_1, but the conditional density functions for q_2 and q_3 may depend on both previous decisions and previous realisations, as in Sprinkle (2000) and Dobbs and Miller (2009). If the 'decision' at any stage is multi-dimensional, then this feature can be accommodated in (8) by thinking of the a_i as vectors of decisions. Thus n, the

number of stages is determined by the number of awards of q_i rather than the number of decisions or tasks.

RPIP implementations with and without bundling

RPIP is most commonly implemented by running a separate lottery for each award of q_i, $i = 1,...,n$, with exchange functions, $G_i(q_i)$, $i = 1,...,n$. In this case, the set of rewards, $\mathbf{q} = (q_1,...,q_n)$, is effectively partitioned into n distinct subsets. However, *RPIP* can be, and has been, implemented differently (with *m<n* lotteries and $\mathbf{q} = (q_1,...,q_n)$ accordingly partitioned into *m* distinct subsets). Partitioning the set q_i, $i = 1,...,n$ into *m<n* partitions for the purpose of implementing *RPIP* is referred to in what follows as 'bundling'. For example, Selten *et al* (1999) partitioned the set $\mathbf{q} = (q_1,...,q_{50})$ into *m*=25 subsets, bundling together pairs of consecutive consequences, whilst Frederickson and Waller (2005) used a single lottery after a forty stage game, bundling together all forty consequences. In subsequent analysis, in common with typical experimental practice, it is assumed that the realized value of q_i is revealed to the subject at the end of each stage *i*.

The objective of RPIP in sequential multi-task single-subject experiments

For the single-task scenario, the objective is mathematically well-defined: to induce a pre-specified preference ordering, denoted here as $v(q)$. The *RPIP* procedure amounts to setting $v(q) = G(q)$, and subjects then behave *as if* maximising the expected value of $G(q)$. In the multi-task context, inductive analysis of how *RPIP*

has *actually* been employed suggests that, for each stage of the experiment, researchers seek to induce a preference ordering for *q* that is *independent* of *q* realized in all other stages of the sequence; that is, in the *i*th stage, the objective is to induce a preference ordering $v_i(q_i)$, that is independent of all $q_j, j \neq i$. Independent preference orderings allow considerable flexibility to an experimenter, not least because there is no requirement for induced preferences to be identical in every stage. But one restriction *is* required. It involves the structure of preferences for the full set of experimental consequences **q**. It is well known that an additively-separable preference function is sufficient for independent preference orderings over each individual consequence, but Koopmans (1972) has also demonstrated it is necessary. Given this, attention can be restricted to the class of additively-separable induced preference functions over the full set of consequences **q**; that is,

$$v(\mathbf{q}) = \sum_{i=1}^{n} v_i(q_i) \tag{9}$$

so that the induced preference function under uncertainty is

$$E\{v(\mathbf{q})\} = \sum_{i=1}^{n} E\{v_i(q_i)\} \tag{10}$$

A dynamic programming approach to maximize (10) involves, at every stage *j*, the subject selecting the action a_j that maximizes $\sum_{i=j}^{n} E_j\{v_i(q_i)\}$, where the subscript on the expectations operator indicates that conditional expectations are taken at stage *j*, as a function of previous actions and realisations of *q*.

All implementations of *RPIP* used in the literature are members of the class of induced preferences examined here. To illustrate, for the frequently-observed case

of $m=n$ lotteries, set $v_i(q_i) = G_i(q_i)$, giving $v(\mathbf{q}) = \sum_{i=1}^{n} G_i(q_i)$; for the Selten *et al*

(1999) case where $m<n$ lotteries, set $v_i(q_i) = q_i$ so that

$$v(\mathbf{q}) = \sum_{i=1}^{50} q_i = \sum_{k=1}^{25} (q_{2k-1} + q_{2k}) = \sum_{k=1}^{25} G_k(q_{2k-1}, q_{2k})$$; for the Fredrickson and Waller

(2005) case where $m=1$, set $v(\mathbf{q}) = G(\mathbf{q})$.

The validity of RPIP with full bundling (m=1)

RPIP is valid for all sequential multi-task single-subject experiments as long as

multiple consequences are fully bundled into a single binary lottery. Proposition 1

below can be proved using the same assumptions as the single-task case presented

above:

> **Proposition 1**: Regardless of personal preferences over money
>
> rewards, as long as the q consequences at each stage are bundled into a
>
> single lottery, $G(\mathbf{q})$, then a *VNM* maximizer will behave as if she has
>
> preferences over $\mathbf{q}$, $G(\mathbf{q})$, pre-specified by the experimenter.

> **Proof**: Available from the authors.

This result is significant firstly because it lends support to the idea that *RPIP* can be

applied to *any* experiment, and secondly, because it offers a 'failsafe' method for

implementing *RPIP* when other bundling solutions, $m>1$, do not work. In the next

section, it is shown that for *RPIP* to be valid without complete bundling, $m>1$, the

class of sequential multi-task single-subject experiments to which it can be applied

must be restricted. For such excluded cases, full bundling, though seldom applied in practice, offers a solution.

3. Stage Independence and *RPIP* without Bundling

This section focuses on the most frequently observed case of 'no bundling' (*m=n*), where $v(\mathbf{q}) = \sum_{i=1}^{n} G_i(q_i)$. In this case, with *n* lotteries, there are *n* money awards; for each stage played, the associated award is paid immediately on completion of the stage. The rewards at different stages are denoted $x_1, x_2, ..., x_n$ and subject preferences are described by an arbitrary *VNM* utility function $U(x_1, x_2, ..., x_n)$.

> **Definition 1:** *Stage independence* is defined as a condition in which (8) can be multiplicatively decomposed and written as:[2]
>
> $$f(\mathbf{q} \mid \mathbf{a}) = f_1(q_1 \mid a_1) \times f_2(q_2 \mid a_2) \times ... \times f_n(q_n \mid a_n) \tag{11}$$
>
> or equivalently $f_i(q_i \mid a_i, q_{i-1}, a_{i-1}, ..., q_1, a_1) = f_i(q_i \mid a_i)$ for all *i*.

> **Proposition 2**: For an arbitrary *n*-stage (*n*≥2) experiment, where stages are not bundled, then regardless of personal preferences over the set of money rewards, a *VNM* maximizer will behave *as if* she has preferences over **q**, $v(\mathbf{q}) = \sum_{i=1}^{n} G_i(q_i)$ *if and only if* the experimental design exhibits stage independence.

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[2] This is a stronger requirement than 'statistical independence'. Statistical independence would merely allow (8) to be multiplicatively decomposed and written as:
$$f(\mathbf{q} \mid \mathbf{a}) = f_1(q_1 \mid a_1) \times f_2(q_2 \mid a_2, a_1) \times ... \times f_n(q_n \mid a_n, a_{n-1}, ..., a_1)$$

Proof: Available from the authors.

Proposition 1 and Proposition 2 have immediate relevance to intermediate cases of bundling, $1<m<n$; for any interdependence within a subset of stages leads to a failure of the necessary condition and, in turn, Proposition 1 indicates a design solution to restore the validity of *RPIP* for that subset of stages. Otherwise, if stages are independent, then the bundling decision is irrelevant to the validity of *RPIP*, leaving researchers with some degree of freedom in how *RPIP* is implemented.

4. Conclusions and Implications for the design of Experiments

Bundling has been incorporated into published *RPIP* designs in diverse ways, yet its purpose has *never* been discussed. We know of no theoretical material referring to bundling, or explaining how it should fit into experimental designs, and this motivates the present article. Berg *et al* (1986) established the validity of *RPIP* for a single risky decision. However, the technique has found widespread use in applications involving multiple stages, multiple agents, and/or complex learning environments. The validity of *RPIP* in these various implementations is thus unclear. The present article establishes conditions for the valid use of *RPIP* – specifically, independence between the multiple stages of the experiment is necessary and sufficient for the most widely-used implementation of binary lotteries, involving a one-to-one correspondence of stages to lotteries, a design described here as 'no bundling'. With interdependence, it is shown that the affected stages must be bundled together into a single lottery.

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